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**HIGGS-RADION SIMILARITY IN PRODUCTION PROCESSES  
INVOLVING OFF-SHELL FERMIONS**

MSU-SINP Preprint N 2014-2/886

Moscow 2014

UDC 539.12.01

PACS: 12.60.-i

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## **HIGGS-RADION SIMILARITY IN PRODUCTION PROCESSES INVOLVING OFF-SHELL FERMIONS**

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Abstract

*The appearance of the radion field and of the radion, the corresponding lowest Kaluza-Klein (KK) mode, is a generic prediction of stabilized brane world models. In such models the radion plays the role of the dilaton, and its mass may be somewhat smaller than that of all the KK modes of other particles propagating in the multidimensional bulk. Due to its origin, the radion couples to the trace of the energy-momentum tensor of the Standard Model, the interaction Lagrangian of the radion and the Standard Model fermions being similar to that of the SM Higgs-fermion interactions except for additional terms, which come into play only in the case of off-shell fermions. In the present paper it is shown that all the contributions due to these additional terms to perturbative amplitudes of physical processes with emitted single radion and an arbitrary number of gauge bosons are canceled out for both massless and massive off-shell fermions. Thus in this case the additional fermion-radion terms in the interaction Lagrangian do not alter any production and decay properties of the radion compared to those of the Higgs boson.*

**Э. Боос, С. Кейзеров, Э. Рахметов, К. Свирина**

## **СХОДСТВО ПРОЦЕССОВ РОЖДЕНИЯ И РАСПАДА РАДИОНА И БОЗОНА ХИГГСА С УЧАСТИЕМ ФЕРМИОНОВ ВНЕ МАССОВОЙ ПОВЕРХНОСТИ**

Препринт НИИЯФ МГУ № 2014-2/886

Аннотация

*Радион – скалярная частица, возникающая в моделях “мира на бране” и взаимодействующая со следом тензора энергии-импульса Стандартной модели (СМ). Лагранжиан взаимодействия радиона с фермионами СМ отличается от лагранжиана взаимодействия бозона Хиггса для фермионов вне их массовой поверхности. Показано, что все дополнительные по сравнению с бозоном Хиггса вклады в амплитуды процессов рождения и распада радиона, в которых участвуют фермионы вне массовой поверхности, взаимно сокращаются как для безмассовых, так и для массивных фермионов. Таким образом, дополнительные члены в лагранжиане взаимодействия не вносят различий в свойства таких процессов для радиона и для бозона Хиггса за исключением общих нормировочных факторов. Указанное сходство является следствием калибровочной инвариантности для процессов с рождением калибровочных бозонов.*

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# 1 Introduction

There are few generic predictions in brane world models as possible extensions of the Standard Model (SM). The fields propagating in the multidimensional bulk manifest themselves as KK towers of states on the brane, where we are supposed to live. These KK tower of states are an example of such generic predictions. Another phenomenon of this kind is the presence of a new scalar field (or fields) on "our" brane (TeV-brane) associated with spin-0 fluctuations of the metric component corresponding to extra space dimension. The size of the extra space dimension should be stabilized in order to get a physically meaningful picture. This leads to the violation of the dilatation invariance, and the spin-0 component of metric fluctuations together with the fluctuations of the additional scalar field introduced for stabilizing the size renders a new scalar field called the radion field, which plays the role of the dilaton in the system at hand.

The radion field is the collection of the spin-0 degrees of freedom, coming from the scalar fluctuations of the metric in extra dimension and of the stabilizing scalar field [1]. For both minimal and non-minimal couplings of the stabilizing scalar field to gravity, this field can be identified with certain combinations of the fluctuations of the metric and of the scalar field in convenient gauges, which connect the fluctuations of the metric with those of the scalar field and coincide with a gauge for isolating the radion field in the unstabilized RS model, if the stabilizing scalar field is put equal to zero [2, 3].

The Randall-Sundrum setup [4] with stabilizing scalar field as proposed by Goldberger and Wise [5] and worked out by DeWolfe, Freedman, Gubser and Karch [6], taking into account the backreaction by solving exactly the coupled equations for the metric and the scalar field, provides an example of a concrete realization of a stabilized brane world model. In a number of studies it was argued that the lowest scalar state, usually called the radion [7, 8, 9], might be significantly lighter than all the other KK excitations [1, 10].

The phenomenology of the radion follows from the simple effective Lagrangian

$$L = -\frac{r(x)}{\Lambda_r} T_\mu^\mu, \quad (1)$$

where  $T_\mu^\mu$  is the trace of the SM energy-momentum tensor,  $r(x)$  stands for the radion field, and  $\Lambda_r$ <sup>1</sup> is a dimensional scale parameter. Since the radion plays the role of the dilaton, it interacts with the massless photon and gluon fields via the well-known dilatation anomaly in the trace of the energy momentum tensor. At the lowest order in the SM couplings the trace has the following form, where the fields, as supposed in most of the studies, are taken on the mass-shell,

$$T_\mu^\mu = \frac{\beta(g_s)}{2g_s} G_{\rho\sigma}^{ab} G_{ab}^{\rho\sigma} + \frac{\beta(e)}{2e} F_{\rho\sigma} F^{\rho\sigma} - m_Z^2 Z^\mu Z_\mu - 2m_W^2 W_\mu^+ W^{-\mu} + \sum_f m_f \bar{f} f, \quad (2)$$

and the sum is taken over all SM fermions,  $\beta(g_s)$  and  $\beta(e)$  are the well-known QCD and QED  $\beta$ -functions.

The interaction vertices of the radion with the SM fields are similar to those of the SM Higgs boson except for the anomaly enhanced interactions with gluons and photons.

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<sup>1</sup>For the parameter  $\Lambda_r$ , the notation  $\Lambda_\phi$  is also often used.  $\Lambda_r$  and another often used parameter  $\Lambda_\tau$  are related by  $\Lambda_r = 2\sqrt{3}\Lambda_\tau$ .

This leads to the corresponding relative enhancement in gluon and photon decay modes and the gluon-gluon fusion production channel of the radion. Various aspects of the decay and production properties of the radion have been touched upon in a number of studies, including a possible mixing of the radion with the Higgs boson, which is important for the radion and Higgs phenomenology [1], [11] - [18].

After the discovery of the Higgs-like boson at the LHC [19] few studies have been carried out investigating a possible impact of the radion as the second scalar particle on the interpretation of the observed boson [20] - [26]. The details depend on particular assumptions of the scenario under consideration, such as which fields propagate in the bulk and which are localized on the brane, what is the Higgs-radion mixing mechanism and what is the mixing parameter etc. But generically the light radion with mass below or above the observed 126 GeV boson is still not fully excluded by all the EW precision constrains and the LHC data.

The fermion part of Lagrangian (1) for on-shell fermions has a very simple form

$$L = - \sum_f \frac{r}{\Lambda_r} m_f \bar{f} f, \quad (3)$$

being the same as for the Higgs boson with the replacement of the scale  $\Lambda_r$  by the Higgs vacuum expectation value,  $\Lambda_r \rightarrow v$ . However, for the case of off-shell fermions Lagrangian (3) needs to be modified. The corresponding Lagrangian can be written symbolically in the following form (the explicit form of the Lagrangian is given in Appendix 1) [27]

$$L = - \sum_f \frac{r}{\Lambda_r} \left[ \frac{3i}{2} ((D_\mu \bar{f}) \gamma^\mu f - \bar{f} \gamma^\mu (D_\mu f)) + 4m_f \bar{f} f \right] + \dots \quad (4)$$

where  $D_\mu$  are the SM covariant derivatives. Note that Lagrangian (4) is non-trivial even for massless fermions.

The aim of this short study is to investigate a possible influence of these additional terms on the processes with tree level diagrams involving the radion and off-shell fermion lines. The extra terms in the Lagrangian lead to a momentum dependence in the vertices potentially giving additional contributions if virtual fermions participate in the process. For the main radion decay processes the fermions are on-shell, and these additional terms in the vertices vanish, but this might not be the case for the radion production.

However, it will be explicitly demonstrated that due to the gauge invariance all the additional contributions to the amplitudes of physical processes are canceled out, and the result for the matrix element is the same as computed with only Higgs-like Lagrangian (3). One should stress that in order to get the mentioned cancellations the trace  $T_\mu^\mu$  of the SM energy momentum tensor should be computed including the SM covariant derivatives as given in Appendix 1. The corresponding Feynman rules containing new 4-point vertices are presented in Appendix 2.

We begin with a simple example of the radion production in the radion-strahlung process. Then we give a more general explanation demonstrating how the cancellation takes place and show a few more examples for the main radion production processes at the LHC and at a Linear Collider.

## 2 The radion-strahlung as a simple example

We begin with a very simple example, the radion production in association with  $Z$  boson in  $e^+e^-$  collisions called the radion-strahlung by analogy with the well-known Higgs production process, the Higgs-strahlung. Even neglecting the very small electron mass there are four contributing Feynman diagrams at the lowest order shown in Fig.1.

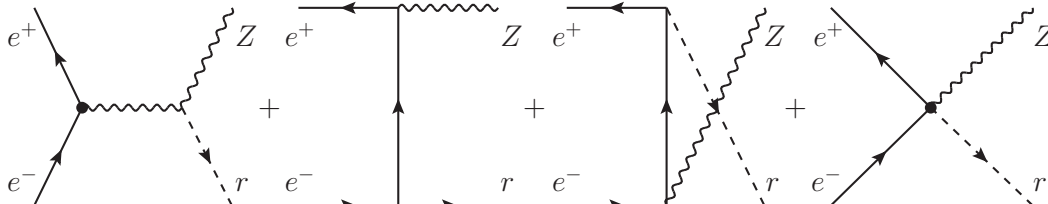


Figure 1: Feynman diagrams contributing to the radion production in association with the  $Z$  boson in  $e^+e^-$  collisions (the radion-strahlung process)

Note that the presence of the last 4-point diagram is a consequence of the gauge invariance. The Feynman rules for the vertices involved are given in Appendix 2. Correspondingly the diagrams give the following contributions to the process amplitude

$$M_1 = -2i C \bar{e}^+(p_2) \Gamma_\mu e^-(p_1) \frac{1}{p^2 - M_Z^2} M_Z^2 \varepsilon^\mu(p_Z) r(p_r) \quad (5)$$

$$M_2 = -i C \bar{e}^+(p_2) \left[ \frac{3}{2} (\not{k} + \not{p}_2) \right] \frac{\not{k}}{k^2} \Gamma_\mu e^-(p_1) \varepsilon^\mu(p_Z) r(p_r) \quad (6)$$

$$M_3 = -i C \bar{e}^+(p_2) \Gamma_\mu \frac{\not{q}}{q^2} \left[ \frac{3}{2} (\not{q} - \not{p}_1) \right] e^-(p_1) \varepsilon^\mu(p_Z) r(p_r) \quad (7)$$

$$M_4 = +3i C \bar{e}^+(p_2) \Gamma_\mu e^-(p_1) \varepsilon^\mu(p_Z) r(p_r), \quad (8)$$

where  $C = \frac{1}{\Lambda_r} \frac{e}{2 \sin \theta_W \cos \theta_W}$ ,  $p = p_1 + p_2 = p_r + p_Z$ ,  $\Gamma_\mu = \gamma_\mu (2 \sin^2 \theta_W - \frac{1-\gamma_5}{2})$ ,  $k = p_2 - p_r$ ,  $q = p_r - p_1$ .

Keeping in mind the Dirac equation  $\not{p}_j e(p_j) = 0$  and the equality  $\not{q} \frac{\not{q}}{q^2} = 1$  it is easy to see that the sum of two diagrams D2 and D3 is exactly canceled out by the last diagram D4, so

$$M_2 + M_3 + M_4 = 0. \quad (9)$$

Therefore the matrix element squared  $|M|^2 = |M_1|^2$  for the radion production is exactly equal to that for the Higgs boson, and the cross section takes the well-known form with the replacement  $\Lambda_r \rightarrow v$  and  $M_r \rightarrow M_h$

$$\sigma(e^+e^- \rightarrow rZ) = \frac{M_Z^2}{\Lambda_r^2} \frac{\alpha(8 \sin^4 \theta_W - 4 \sin^2 \theta_W + 1)}{24 \sin^2 \theta_W \cos^2 \theta_W} \frac{\sqrt{\lambda_r}}{4s^2} \frac{\lambda_r + 12M_Z^2 s}{(s - M_Z^2)^2}, \quad (10)$$

where  $\lambda_r = (M_Z^2 + M_r^2 - s)^2 - 4M_Z^2 M_r^2$ .

As one can see all the additional contributions are canceled out. The same property of the cancellation of additional to the Higgs-like contributions takes place for the associated radion and  $W^\pm$  boson production, for example,

$$u\bar{d} \rightarrow rW^+ \quad (11)$$

where there are also 4 contributing diagrams similar to those in Fig.1.

### 3 Cancellations of additional to the Higgs-like contributions in tree-level amplitudes

The observed cancellation follows from the structure of the fermion current with the emission of the radion and a number of gauge bosons.

Let us begin with a simple case with just one gauge boson emission as shown in Fig.2. It corresponds to the radion-strahlung process considered in the previous section with Z boson emission. However the cancellation property is valid for any emitted SM gauge boson and for any massive SM fermion.

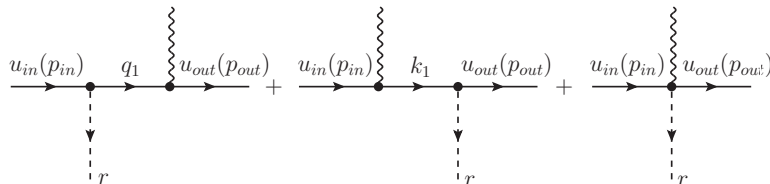


Figure 2: *Fermion current radiating the radion and an SM vector gauge boson*

In order to see that it is instructive to rewrite the fermion-radion vertex in the following way

$$\begin{aligned} & \frac{i}{\Lambda_r} \left[ \frac{3}{2} (\not{p}_{out} + \not{p}_{in}) - 4 m_f \right] = \\ & \frac{i}{\Lambda_r} \left[ \frac{3}{2} (\not{p}_{out} - m_f) + \frac{3}{2} (\not{p}_{in} - m_f) - m_f \right] = \\ & \frac{i}{\Lambda_r} \left[ \frac{3}{2} S^{-1}(p_{in}) + \frac{3}{2} S^{-1}(p_{out}) - m_f \right], \end{aligned} \quad (12)$$

where  $S^{-1}(p)$  is the inverse function to the propagator <sup>2</sup>

$$S(p) = \frac{\not{p} + m_f}{p^2 - m_f^2}.$$

Note that the last term proportional to  $m_f$  in vertex (12) is exactly the same as for the Higgs boson with the obvious replacement  $\Lambda_r \rightarrow v$ .

The diagrams in Fig.2 can be expressed as follows <sup>3</sup>

$$D_1 = -i C \bar{u}_{out}(p_{out}) \Gamma_\mu S(k_1) \left[ \frac{3}{2} (S^{-1}(k_1) + S^{-1}(p_{in})) - m_f \right] u_{in}(p_{in}) \quad (13)$$

$$D_2 = -i C \bar{u}_{out}(p_{out}) \left[ \frac{3}{2} (S^{-1}(p_{out}) + S^{-1}(q_1)) - m_f \right] S(q_1) \Gamma_\mu u_{in}(p_{in}) \quad (14)$$

$$D_3 = +i 3 C \bar{u}_{out}(p_{out}) \Gamma_\mu u_{in}(p_{in}) \quad (15)$$

where the constant C includes the product of all the factors of the vertices involved,  $\Gamma_\mu$  is the Lorentz part of the fermion-boson and fermion-boson-radion vertices, which is

<sup>2</sup>All the relevant constants from propagators and vertices are included into the common factors of the amplitudes.

<sup>3</sup>To make expressions shorter in all formulas below for the fermion currents obvious Lorentz indexes are omitted in the left hand sides

the same in two vertices coming from the same covariant derivative. The initial and the final spinors correspond to the same fermion for the neutral fermion current and they are different for the charged current. Once more one can easily see that using the Dirac equation for *in* and *out* fermion states and summing up  $D_1 + D_2 + D_3$  the only remaining contribution is proportional to the fermion mass and is the same as that for the Higgs boson.

This property of the cancellation of all the contributions additional to the Higgs-like part can be generalized to an arbitrary number, say  $N$ , of gauge bosons emitted from the fermion line in association with the radion as shown in Fig.3.

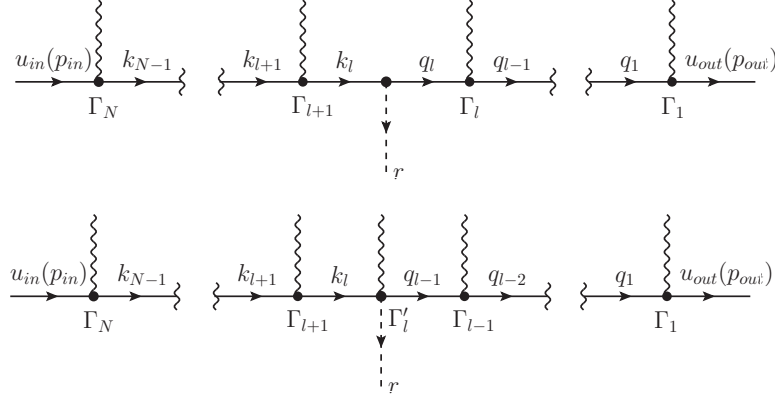


Figure 3: Fermion current radiating the radion and  $N$  SM vector gauge bosons in a three-point vertex (upper graph) and in a four-point vertex (lower graph) corresponding to the contributions  $M^l$  and  $M'_l$  respectively.

The sum of all diagrams can be split into parts containing the fermion-fermion-radion vertex and the boson-fermion-fermion-radion vertex

$$M_{N \text{ vector bosons}} = M_0 + \sum_{l=1}^N (M_l + M'_l), \quad (16)$$

where

$$M_l \sim i^{2N+1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^l \Gamma_{\mu_j}^j S(q_j) \right] \left[ -\frac{3}{2} (S^{-1}(q_l) + S^{-1}(k_l)) + m_{f_l} \right] \left[ \prod_{j=l+1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}), \quad (17)$$

for  $l = 1, \dots, N - 1$

$$M_0 \sim i^{2N+1} \bar{f}_{out}(p_{out}) \left[ -\frac{3}{2} (S^{-1}(p_{out}) + S^{-1}(k_0)) + m_{f_{out}} \right] \left[ \prod_{j=1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}), \quad (18)$$

$$M_N \sim i^{2N+1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^N \Gamma_{\mu_j}^j S(q_j) \right] \left[ -\frac{3}{2} (S^{-1}(q_N) + S^{-1}(p_{in})) + m_{f_{in}} \right] f_{in}(p_{in}), \quad (19)$$

$$M'_l \sim i^{2N-1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^{l-1} \Gamma_{\mu_j}^j S(q_j) \right] [-3 \Gamma_{\mu_l}^l] \left[ \prod_{j=l+1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}), \quad (20)$$

for  $l = 2, \dots, N-1$

$$M'_1 \sim i^{2N-1} \bar{f}_{out}(p_{out}) [-3 \Gamma_{\mu_1}^1] \left[ \prod_{j=2}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}) \quad (21)$$

$$M'_N \sim i^{2N-1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^{N-1} \Gamma_{\mu_j}^j S(q_j) \right] [-3 \Gamma_{\mu_N}^N] f_{in}(p_{in}), \quad (22)$$

$\Gamma_{\mu_j}^j$  is the SM fermion and gauge boson vertex,  $k_l = p_{in} - \sum_{j=l}^N p_j^V$ ,  $q_0 = p_{in}$ ,  $q_l = k_l - p_r = p_{out} + \sum_{j=1}^l p_j^V$ ,  $p_j^V$  is the vector boson momentum, the upper index  $V$  standing for any SM gauge vector boson (Z, W, photon, gluon),  $p_r$  is the radion momentum,  $m_{f_l}$ ,  $m_{f_{in}}$ ,  $m_{f_{out}}$  are the masses of the internal, initial and final state fermions which may be different in the case of the charged current. Note that the factors  $i^{2N+1}$  and  $i^{2N-1}$  are different due to the difference by one of the number of the propagators and vertices involved into expressions (17), (18), (19) and (20), (21), (22). This leads to a relative  $(-1)$  sign between the two contributions.

From formulas (17)–(22) one can write simple relations taking into account the relation  $S^{-1}(q_j) \times S(q_j) = 1$  and the Dirac equations for *in* and *out* fermions

$$M_l = M_l^H - \frac{1}{2} M'_l - \frac{1}{2} M'_{l+1} \quad (23)$$

$$M_0 = M_0^H - \frac{1}{2} M'_1 \quad (24)$$

$$M_N = M_N^H - \frac{1}{2} M'_N, \quad (25)$$

where the Higgs-like contributions  $M_0^H$ ,  $M_N^H$ ,  $M_l^H$  labeled with the  $H$  symbol being proportional to the fermion masses have the form which follows from (17),(18),(19)

$$M_l^H \sim i^{2N+1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^l \Gamma_{\mu_j}^j S(q_j) \right] [m_{f_l}] \left[ \prod_{j=l+1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}), \quad (26)$$

$$M_0^H \sim i^{2N+1} \bar{f}_{out}(p_{out}) [m_{f_{out}}] \left[ \prod_{j=1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] f_{in}(p_{in}), \quad (27)$$

$$M_N^H \sim i^{2N+1} \bar{f}_{out}(p_{out}) \left[ \prod_{j=1}^N \Gamma_{\mu_j}^j S(q_j) \right] [m_{f_{in}}] f_{in}(p_{in}), \quad (28)$$

Summing up the left and the right hand sides of equation (23) from 1 to  $N-1$  and adding the left and the right hand sides of equations (24) and (25) one gets the following equality

$$\sum_{l=0}^N M_l + \sum_{l=1}^N M'_l = \sum_{l=0}^N M_l^H \quad (29)$$

The result in formula (29) means that the sum of all the contributions leads to only the Higgs-like type of the contribution and all the other parts are canceled out explicitly.



## 4 Generalization to the loop case

The main result of the previous section can be easily generalized to the loop case. First of all, one should mention that in the way of proving equality (29) nothing special was required for boson lines in the vertices. The boson lines in Fig.3 could correspond to real particles or virtual propagators in the loops. In this sense all the above considerations are valid for both real and virtual gauge bosons emitted from the fermion current.

In the case of a fermion loop one can follow a similar logic as was given above. In the loop case there is an additional fermion propagator instead of the external spinors at the tree level. Correspondingly the expressions for various fermion loop contributions with emission of  $N$  gauge bosons have the following form by analogy with (17),(19) and (20),(21),(22).

$$M_l \sim i^{2N+1} Tr \left\{ \left[ \prod_{j=1}^l \Gamma_{\mu_j}^j S(q_j) \right] \left[ -\frac{3}{2} (S^{-1}(q_l) + S^{-1}(k_l)) + m_{f_l} \right] \right. \\ \left. \left[ \prod_{j=l+1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] S(p) \right\}, \quad (30)$$

for  $l = 1, \dots, N-1$ ,

$$M_N \sim i^{2N+1} Tr \left\{ \left[ \prod_{j=1}^N \Gamma_{\mu_j}^j S(q_j) \right] \left[ -\frac{3}{2} (S^{-1}(q_N) + S^{-1}(p_{in})) + m \right] S(p) \right\}, \quad (31)$$

$$M'_l \sim i^{2N-1} Tr \left\{ \left[ \prod_{j=1}^{l-1} \Gamma_{\mu_j}^j S(q_j) \right] \left[ -3 \Gamma_{\mu_l}^l \right] \left[ \prod_{j=l+1}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] S(p) \right\}, \quad (32)$$

for  $l = 2, \dots, N-1$

$$M'_1 \sim i^{2N-1} Tr \left\{ \left[ -3 \Gamma_{\mu_1}^1 \right] \left[ \prod_{j=2}^N S(k_{j-1}) \Gamma_{\mu_j}^j \right] S(p) \right\}, \quad (33)$$

$$M'_N \sim i^{2N-1} Tr \left\{ \left[ \prod_{j=1}^{N-1} \Gamma_{\mu_j}^j S(q_j) \right] \left[ -3 \Gamma_{\mu_N}^N \right] S(p) \right\}. \quad (34)$$

where the momenta are expressed as  $k_l = p - \sum_{j=l}^N p_j^V$ ,  $q_l = k_l - p_r = p + \sum_{j=1}^{l-1} p_j^V$ . Note that the contribution  $M_0$  is not present now since it coincides with  $M_N$ .

In the same manner as in the previous section we get

$$M_l = M_l^H - \frac{1}{2} M'_l - \frac{1}{2} M'_{l+1} \quad (35)$$

$$M_N = M_N^H - \frac{1}{2} M'_N - \frac{1}{2} M'_1, \quad (36)$$

From relations (35) and (36) it is easy to show that

$$\sum_{l=1}^N M_l = \sum_{l=1}^{N-1} M_l + M_N = \sum_{l=1}^N M_l^H - \sum_{l=1}^N M'_l \quad (37)$$

and therefore one gets once again the equality

$$\sum_{l=1}^N M_l + \sum_{l=1}^N M'_l = \sum_{l=1}^N M_l^H, \quad (38)$$

which demonstrates that all the contributions except for the Higgs-like type are canceled out in the case of a fermion loop.

## 5 Conclusions

In all the main radion production processes, there are contributing Feynman diagrams involving off-shell fermions. The radion is emitted from various fermion currents containing off-shell fermion propagators in the radion-strahlung or the vector boson fusion production in  $e^+e^-$  collisions, as well as in the vector boson fusion, associated radion and vector boson production, associated radion and the top-quark pair production processes in hadron collisions at the LHC. The off-shell fermions participate in fermion loops for gluon-gluon fusion production process at the LHC and in  $gg, \gamma\gamma, \gamma Z$  decay modes of the radion. The additional to the Higgs boson case non-trivial vertices (even for massless fermions) of fermion-radion interactions follow from the structure of the trace of the gauge invariant SM energy-momentum tensor as given in Appendices 1,2.

We have shown that all the contributions to perturbative amplitudes of physical processes with a single radion are canceled out due to the corresponding additional terms for both massless and massive off-shell fermions. We demonstrated, first, the cancellation in a simple example of the radion-strahlung process in  $e^+e^-$  collisions. Then we presented a general proof of the cancellation for an arbitrary fermion current radiating a single radion and any number of the SM gauge bosons. This proof was generalized to the case of the amplitudes containing closed fermion loops and an arbitrary number of the gauge bosons.

The proof also means that, in calculating the amplitudes of processes with a single radion, the terms with the covariant derivatives of the fermion fields in Lagrangian (4) can be replaced by the mass terms in accordance with the equation of motion for the fermion fields, which is not *a priori* justified in the gauge theory, where the quantization is performed with the help of the path integration. Thus the additional fermion-radion terms in the interaction Lagrangian do not alter any production and decay properties of a single radion compared to those of the Higgs boson.

It is worth noting that the observed cancellation property, being a result of the gauge invariant structure of the SM energy-momentum trace, is also valid for any scalar particle (not only the radion) which interacts with the SM particles via the trace.

In our study we concentrated on physical processes with emitted single radion and an arbitrary number of gauge bosons. In the case of the radion and the Higgs boson emission there are differences in comparison with the Higgs boson pair production. In the radion and Higgs case the contributions due to off-shell fermions do not cancel each other leading to remaining terms which are absent in the Higgs pair production case. Another difference comes from the diagrams involving the triple  $hhr$  vertex. As shown in Appendix 2, the vertex  $hhr$  contains an additional, compared to the  $hhh$  vertex, momentum depending part. As for the radion pair production, the amplitude of the

process is of the order  $\Lambda_r^{-2}$ . In this case some contributions come from corrections to the interactions of the radion and the SM fields of the same  $\Lambda_r^{-2}$  order, such as the two fermions - two radions vertex.

However a realistic analysis of this pair production processes in experiments at colliders, particularly at the LHC, is problematic. The Higgs boson pair production process has a rather small rate even at the LHC, and a production process leading to various final states is difficult to be extracted from the backgrounds (see, e.g., [28]). Taking into account that the rate of the pair production processes involving the radion is even smaller than the rate of the Higgs boson pair production, a delicate analysis including backgrounds is needed to understand the influence and the impact of the mentioned differences between the processes on their observability.

An interesting question arises whether the observed cancellations take place in other variants of brane world models, in particular, when the SM fields are allowed to propagate in the bulk leading to the appearance of the corresponding KK-towers of the fermion and boson states. In this case the interaction Lagrangian of the radion and the SM fields is more complicated, because it includes a contribution from the energy-momentum tensor in the extra dimension [18]. The structure of this interaction is different from the one, when the radion couples to the SM energy-momentum trace in four dimensions. Thus, the cancellation of the off-shell fermion contributions including the fermion KK-towers is not obvious, and the corresponding analysis deserves a separate study.

## 6 Acknowledgments

The work was supported by grant 14-12-00363 of Russian Science Foundation. The authors are grateful to M. Smolyakov and I. Volobuev for useful discussions and critical remarks.

## 7 Appendix 1

### Trace of the SM energy-momentum tensor.

The trace of the SM energy-momentum tensor calculated as the variation of the SM Lagrangian with respect to the metric [29, 30] can be written in the unitary gauge as follows:

$$\begin{aligned}
T_\mu^\mu &= -(\partial_\mu h)(\partial^\mu h) + 2m_h^2 h^2 \left(1 + \frac{h}{2v}\right)^2 - 2m_W^2 W_\mu^+ W^\mu - \left(1 + \frac{h}{v}\right)^2 - m_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2 \\
&+ \sum_f \left\{ -\frac{i3}{2} [\bar{f}\gamma^\mu (\partial_\mu f) - (\partial_\mu \bar{f}) \gamma^\mu f] + 4m_f \bar{f} f \right\} + \frac{4h}{v} \sum_f m_f \bar{f} f - 3e A_\mu \sum_f q_f \bar{f} \gamma^\mu f \\
&- \frac{3m_Z}{2v} Z_\mu \sum_f \bar{f} \gamma^\mu [a_f + b_f \gamma_5] f - \frac{3}{\sqrt{2}} \frac{m_W}{v} (W_\mu^- \bar{\nu}_j U_{jk}^{PMNS} \gamma^\mu [1 - \gamma_5] e_k + h.c.) \\
&- \frac{3}{\sqrt{2}} \frac{m_W}{v} (W_\mu^- \bar{u}_j \gamma^\mu [1 - \gamma_5] V_{jk}^{CKM} d_k + h.c.) - 3g_c (\bar{u}_j \gamma^\mu \hat{G}_\mu u_j + \bar{d}_j \gamma^\mu \hat{G}_\mu d_j) \\
&+ \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu} + \frac{\beta(g_s)}{2g_s} G_{\mu\nu}^{ab} G^{ab\mu\nu},
\end{aligned}$$

where  $U^{PMNS}$  is the Pontecorvo-Maki-Nakagawa-Sakata matrix,  $V^{CKM}$  is the Cabibbo-Kobayashi-Maskawa matrix. The last two terms take the anomalies into account, they describe the interaction of the radion with the photon and the gluon fields.

## 8 Appendix 2

### Interaction vertices of radion and SM fields

Radion and two fermions:

$$r(k_r)\bar{f}(p_{\bar{f}})f(p_f) \quad \frac{i}{\Lambda_r} \left\{ \frac{3}{2} [\not{p}_{\bar{f}} - \not{p}_f] + 4m_f \right\} = \frac{i}{\Lambda_r} \left\{ \frac{3}{2} [(\not{p}_{\bar{f}} + m_f) - (\not{p}_f - m_f)] + m_f \right\}$$

Radion, two fermions and photon:

$$r(k_r)\gamma(k_\gamma)\bar{f}(p_{\bar{f}})f(p_f) \quad -i\frac{3eq_f}{\Lambda_r}\gamma^\mu$$

Radion, two fermions and  $Z$ -boson:

$$r(k_r)Z(k_Z)\bar{f}(p_{\bar{f}})f(p_f) \quad -i\frac{1}{\Lambda_r}\frac{3m_Z}{2v}\gamma^\mu [a_f + b_f\gamma_5]$$

Radion, two fermions and  $W$ -boson:

$$r(k_r)W(k_W)\bar{u}(p_u)d(p_d) \quad -i\frac{1}{\Lambda_r}\frac{3m_W}{\sqrt{2}v}V_{jk}^{CKM}\gamma^\mu [1 - \gamma_5]$$

$$r(k_r)W(k_W)\bar{\nu}(p_\nu)e(p_e) \quad -i\frac{1}{\Lambda_r}\frac{3m_W}{\sqrt{2}v}U_{jk}^{PMNS}\gamma^\mu [1 - \gamma_5]$$

Radion, two fermions and Higgs-boson:

$$r(k_r)h(k_h)\bar{f}(p_{\bar{f}})f(p_f) \quad i\frac{1}{\Lambda_r}\frac{4m_f}{v}$$

Radion and two Higgs-bosons:

$$r(k_r)h(p_1)h(p_2) \quad i\frac{1}{\Lambda_r} \{p_{1\mu}p_2^\mu - 2m_h^2\}$$

Radion and three Higgs-bosons:

$$r(k_r)h(p_1)h(p_2)h(p_3) \quad -i\frac{1}{\Lambda_r}\frac{12m_h^2}{v}$$

Radion and four Higgs-bosons:

$$r(k_r)h(p_1)h(p_2)h(p_3)h(p_4) \quad -i\frac{1}{\Lambda_r}\frac{12m_h^2}{v^2}$$

Radion and two  $Z$ -bosons:

$$r(k_r)Z(p_1)Z(p_2) \qquad -i\frac{2m_Z^2}{\Lambda_r}g^{\mu\nu}$$

Radion, two  $Z$ -bosons and Higgs-boson:

$$r(k_r)Z(p_1)Z(p_2)h(p_3) \qquad -i\frac{1}{\Lambda_r}\frac{4m_Z^2}{v}g^{\mu\nu}$$

Radion, two  $Z$ -bosons and two Higgs-bosons:

$$r(k_r)Z(p_1)Z(p_2)h(p_3)h(p_4) \qquad -i\frac{1}{\Lambda_r}\frac{4m_Z^2}{v^2}g^{\mu\nu}$$

Radion and two  $W$ -bosons:

$$r(k_r)W(p_1)W(p_2) \qquad -i\frac{2m_W^2}{\Lambda_r}g^{\mu\nu}$$

Radion, two  $W$ -bosons and Higgs-boson:

$$r(k_r)W(p_1)W(p_2)h(p_3) \qquad -i\frac{1}{\Lambda_r}\frac{4m_W^2}{v}g^{\mu\nu}$$

Radion, two  $W$ -bosons and two Higgs-bosons:

$$r(k_r)W(p_1)W(p_2)h(p_3)h(p_4) \qquad -i\frac{1}{\Lambda_r}\frac{4m_W^2}{v^2}g^{\mu\nu}$$

## References

- [1] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001) [hep-th/0008151].
- [2] E. E. Boos, Y. S. Mikhailov, M. N. Smolyakov and I. P. Volobuev, Mod. Phys. Lett. A **21** (2006) 1431 [hep-th/0511185].
- [3] A. S. Mikhailov, Y. S. Mikhailov, M. N. Smolyakov and I. P. Volobuev, Theor. Math. Phys. **161** (2009) 1424 [arXiv:0812.2699 [hep-th]].
- [4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].
- [5] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [hep-ph/9907447]; W. D. Goldberger and M. B. Wise, Phys. Lett. B **475**, 275 (2000) [hep-ph/9911457].
- [6] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62** (2000) 046008 [arXiv:hep-th/9909134].
- [7] W. D. Goldberger and M. B. Wise, Phys. Lett. B **475**, 275 (2000) [hep-ph/9911457];

- [8] C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D **62**, 045015 (2000) [hep-ph/9911406].
- [9] C. Charmousis, R. Gregory and V. A. Rubakov, Phys. Rev. D **62**, 067505 (2000) [hep-th/9912160].
- [10] E. E. Boos, Y. S. Mikhailov, M. N. Smolyakov and I. P. Volobuev, Nucl. Phys. B **717**, 19 (2005) [hep-th/0412204].
- [11] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **595**, 250 (2001) [hep-ph/0002178];
- [12] K. -m. Cheung, Phys. Rev. D **63**, 056007 (2001) [hep-ph/0009232];
- [13] G. D. Kribs, eConf C **010630**, P317 (2001) [hep-ph/0110242];
- [14] M. Chaichian, A. Datta, K. Huitu and Z. -h. Yu, Phys. Lett. B **524**, 161 (2002) [hep-ph/0110035].
- [15] T. G. Rizzo, JHEP **0206**, 056 (2002) [hep-ph/0205242];
- [16] D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, Nucl. Phys. B **671**, 243 (2003) [hep-ph/0206192];
- [17] J. F. Gunion, M. Toharia and J. D. Wells, Phys. Lett. B **585**, 295 (2004) [hep-ph/0311219].
- [18] C. Csaki, J. Hubisz and S. J. Lee, Phys. Rev. D **76**, 125015 (2007) [arXiv:0705.3844 [hep-ph]].
- [19] G. Aad et al (ATLAS Collaboration), Phys. Lett. **B716**, 1 (2012); S. Chatrchyan et al (CMS Collaboration), Phys. Lett. **B716**, 30 (2012).
- [20] Z. Chacko, R. Franceschini and R. K. Mishra, JHEP **1304**, 015 (2013) [arXiv:1209.3259 [hep-ph]];
- [21] Z. Chacko, R. K. Mishra and D. Stolarski, JHEP **1309**, 121 (2013) [arXiv:1304.1795 [hep-ph]];
- [22] G. -C. Cho, D. Nomura and Y. Ohno, Mod. Phys. Lett. A **28**, 1350148 (2013) [arXiv:1305.4431 [hep-ph]];
- [23] N. Desai, U. Maitra and B. Mukhopadhyaya, JHEP **1310**, 093 (2013) [arXiv:1307.3765 [hep-ph]];
- [24] P. Cox, A. D. Medina, T. S. Ray and A. Spray, JHEP **1402**, 032 (2014) [arXiv:1311.3663 [hep-ph]];
- [25] M. Geller, S. Bar-Shalom and A. Soni, Phys. Rev. D **89**, 095015 (2014) [arXiv:1312.3331 [hep-ph]];
- [26] D. -W. Jung and P. Ko, Phys. Lett. B **732**, 364 (2014) [arXiv:1401.5586 [hep-ph]].

- [27] E. E. Boos, V. E. Bunichev, M. N. Smolyakov and I. P. Volobuev, Phys. Rev. D **79**, 104013 (2009) [arXiv:0710.3100 [hep-ph]].
- [28] S. Dawson, A. Gritsan, H. Logan, J. Qian, C. Tully, R. Van Kooten, A. Ajajib and A. Anastassov *et al.*, arXiv:1310.8361 [hep-ex]
- [29] L.D. Landau, E.M. Lifshitz, "The Classical Theory of Fields", Ch. 11.94, Pergamon Press, Oxford (1975).
- [30] N.D. Birrell, P.C.W. Davies, "Quantum fields in curved space", Ch. 3.8, Cambridge University Press (1982).

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**СХОДСТВО ПРОЦЕССОВ РОЖДЕНИЯ И РАСПАДА РАДИОНА И БОЗОНА  
ХИГГСА С УЧАСТИЕМ ФЕРМИОНОВ ВНЕ МАССОВОЙ ПОВЕРХНОСТИ**

Препринт НИИЯФ МГУ № 2014-2/886

Работа поступила в ОНТИ *02.10.2014*